A guess of the pebble trajectory parameterised as a fn. of time might be:

(1)

or

(11)

or

(111)

So which is right?

Centre of tyre has position \( \vec{x}_c = (Vt, R) \)

Position of pebble relative to centre:

\[
\vec{x}_{p\text{ rel}c} = (-R\sin(\omega t), -R\cos(\omega t))
\]

(as \( \vec{x}_c = -R\hat{j} \) at \( t = 0 \))

To find \( \omega \):

\[
2\pi R = V T \quad \text{period}
\]

\[
\Rightarrow \frac{2\pi}{T} = \omega = \frac{V}{R}
\]

\[
\Rightarrow V = RW \quad (\text{An important relation})
\]

\[
\Rightarrow \vec{x}_p = \vec{x}_c + \vec{x}_{p\text{ rel}c} = (Vt - R\sin(\frac{Vt}{R}), R(1 - \cos(\frac{Vt}{R})))
\]

\[
\vec{v}_p = (V(1 - \cos(\frac{Vt}{R})), VSin(\frac{Vt}{R}))
\]

\[
\vec{a}_p = \left( \frac{V^2}{R} \sin(\frac{Vt}{R}), \frac{V^2}{R} \cos(\frac{Vt}{R}) \right)
\]

Consider \( (\vec{v}_p)_x \): it is always \( \geq 0 \). Thus, (11) is the correct trajectory.