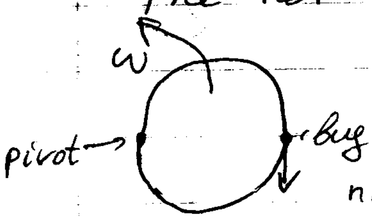
Physics 141

Homework №6

Solutions

№6.3. A ring of mass M and radius R lies on its side on a frictionless table. It's pivoted at its rim. A bug of mass m walks around the ring with speed v, starting at the pivot. What is the rotational velocity of the ring when the bug is a) halfway around, b) back at the pivot.

The net angular momentum is always zero.



At the point of the pivot the angular momentum of the bug is 0, no matter what its speed is. So in part b) the rotational velocity of the ring $\omega = 0$.

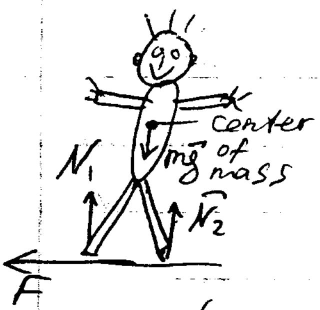
In part a). The velocity of the bug is $(v - 2R\omega)$ and its angular momentum is $2Rm(v - 2R\omega)$. The moment of inertia of the ring is $I = 2MR^2$, so

$$-\omega \cdot \cancel{2}MR^{\cancel{2}} + \cancel{2}\cancel{R}m(v - 2R\omega) = 0$$

$$MR\omega = mv - 2mR\omega$$

$$\omega R(M + 2m) = mv \Rightarrow \omega = \frac{mv}{R(M+2m)}$$

№6.6. A man of mass M stands on a railroad car which is rounding an unbanked turn of radius R at v. His center of mass is L above the car, his feet d apart. How much weight is on each of his feet



The force of friction $F = \frac{mv^2}{R}$.
The net ~~torque~~ torque with respect to the center of mass is:

$$-F \cdot L + N_1 \frac{d}{2} + N_2 \frac{d}{2} = 0, \text{ so}$$

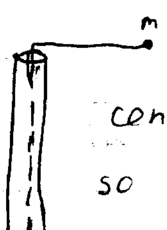
$(N_2 - N_1) = \frac{2FL}{d}$; On the other hand,
$N_1 + N_2 = mg$, so $N_2 = \frac{2mv^2L}{dR} + N_1$,

$$2N_1 + \frac{2mv^2L}{dR} = mg \Rightarrow N_1 = \frac{mg}{2} - \frac{mv^2L}{dR},$$

$$N_2 = \frac{mg}{2} + \frac{mv^2L}{dR}$$

N 6.13. Mass m is attached to a post of radius R by a string. Initially, it's r from the center of the post and is moving tangentially with speed v_0.
In case a) the string passes through a hole.
In case b) the string wraps around. What is conserved? Find the final speed.

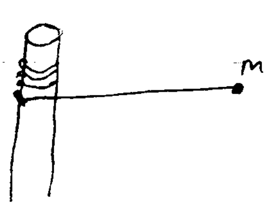
a) The string is pulled, so the work is done and the kinetic energy is not conserved. However, the torque is zero, so <u>the angular momentum is conserved</u>



$m v_0 r = m v_f R \Rightarrow$ the final velocity is

$$\underline{v_f = v_0 \frac{r}{R}}$$

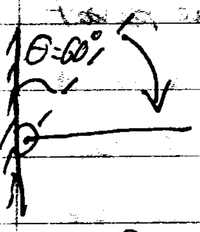
b)



The torque is not zero now (it's $F \cdot R$), so the angular momentum is not conserved. However, where the force is applied (at the surface of the post) the rope is at rest, so no work is done, at the kinetic energy is conserved.

$K = \frac{Mv^2}{2}$, so v is also conserved and $v_f = v_0$

√6.20 A thin plank of mass M and length l is pivoted at one end. It is released at 60° from the vertical. What is the magnitude and direction of the force on the pivot when the plank is horizontal?



The angular momentum of the plank is $L = \frac{1}{3}Ml^2\omega = \frac{1}{3}Ml^2\frac{v}{l/2} =$

$= \frac{2}{3}Mlv$; The torque is $\tau = Mg\frac{l}{2}$ (when $\theta = 90°$), So $\frac{dL}{dt} = \tau \Rightarrow$

$\frac{2}{3}Ml\dot{v} = Mg\frac{l}{2}$, so the acceleration of the center of mass is $\dot{v} = \frac{3}{4}g$. This is the tangential acceleration. It look's downward.

Then the vertical component of the force at the pivot is found from $F_{vert} - Mg = -\frac{3}{4}Mg \Rightarrow$

$\Rightarrow \frac{1}{4}Mg$.

There is also a centripetal acceleration, horizontal and pointing to the left, and equal to $\frac{v^2}{l/2} = \omega^2\frac{l}{2}$. We find ω:

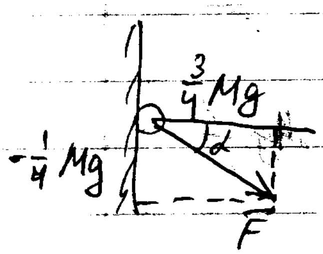
from the conservation of energy: the change of the potential energy is $Mg \cdot \frac{l}{2} \cdot \cos 60° = Mg\frac{l}{4}$. So $\frac{1}{3}Ml^2\frac{\omega^2}{2} = Mg\frac{l}{4} \Rightarrow$

$\omega^2 = \frac{3}{2}\frac{g}{l}$, and the centripetal acceleration

is $\omega^2\frac{l}{2} = \frac{3}{4}g$ (same value!)

Then the horizontal component of the force at the pivot is $\frac{3}{4}Mg$ and points to the left.

We found the force exerted on the plank by the pivot. The force exerted on the pivot is minus that.



$\tan\alpha = \frac{1}{4}\cdot\frac{4}{3} = \frac{1}{3}$, so

$\alpha \simeq 18°$

$|F| = Mg\sqrt{\frac{1}{16}+\frac{9}{16}} = Mg\frac{\sqrt{10}}{4}$

№632 A solid rubber wheel of radius R and mass M rotates with angular velocity ω_0 about a frictionless pivot. A second rubber wheel of radius r and mass m, also mounted on a frictionless pivot, is brought into contact with it. What's the final ω of the first wheel?

The speed of the rim must be the same for both wheels: $\omega_1 R = \omega_2 r$.

Newton's third law tells us that

The forces acting between the wheels, whatever they are, are equal and opposite. Then if the angular impulse (I mean by that $\int$ torq d(time)) acting on the first wheel is S_1, that acting on the second wheel is $-S_1\frac{r}{R}$. We have then:

$\frac{1}{2}MR^2(\omega_1-\omega_0)=S_1$; $\frac{1}{2}mr^2\omega_2=-S_1\frac{r}{R}=$

$=-\frac{r}{R}\cdot\frac{1}{2}MR^2(\omega_1-\omega_0)=\frac{(\omega_0-\omega_1)}{2}MRr.$

From here $mr\omega_2=(\omega_0-\omega_1)MR$

We found in the beginning $\omega_2 r=\omega_1 R$, so

$m\omega_1\cancel{R}=(\omega_0-\omega_1)M\cancel{R}$

$\omega_1=\frac{M}{m+M}\omega_0$. So the answer doesn't depend on R and r!

N6.40 A wheel with fine teeth is attached to the end of a spring with constant k and unstretched length l. For $x>l$, the wheel slips freely on the surface. But for $x<l$ the teeth mesh with the teeth on the ground. All the mass of the wheel is in its rim

a) The wheel is released from $x=l+b$. How close will it come to the wall in its first trip?

b) How far will it go when it leaves the wall

c) What happens when the wheel again hits the gear track?

a) The kinetic energy of the wheel when it doesn't slip is $K = \frac{Mv^2}{2} + \frac{MR^2}{2}\omega^2 =$
$= \frac{Mv^2}{2} + \frac{Mv^2}{2} = \frac{1}{2}(2M)v^2$. So it's like doubling its mass.

The initial energy is $\frac{kb^2}{2}$. The final kinetic energy when the wheel reaches $x = l$ is $\frac{Mv_0^2}{2}$, so $v_0 = b\sqrt{\frac{k}{M}}$. Here it hits the gear track. Some impulse $S = \int F dt$ is acting. Then $M(v_1 - v_0) = S$, and
$MR^2\left(\frac{v_1}{R}\right) = -S \cdot R$ (the change of the momentum and of the angular momentum). So $v_1 = \frac{1}{2}v_0$ (just as if we suddenly double the mass!)
Now the potential energy is 0, and the kinetic $K = Mv_1^2$, so the wheel stops at $x = l - a$, where $\frac{ka^2}{2} = Mv_1^2 = \frac{M}{4}v_0^2 =$
$= \frac{M}{4} b^2 \frac{k}{M} \Rightarrow a^2 = \frac{1}{2} b^2 \Rightarrow a = \frac{b}{\sqrt{2}}$.

$$\underline{x_{final} = l - \frac{b}{\sqrt{2}}}.$$

b) The wheel comes back to $x = l$, having the same velocity v_1. Due to slipping, the rotational part of energy doesn't change, so the conservation of energy says that if $x_{max} = l + c$, $\frac{kc^2}{\cancel{2}} = \frac{Mv_1^2}{\cancel{2}} \Rightarrow$

$$\Rightarrow c = \sqrt{\frac{M}{k}}\, v_1 = \sqrt{\frac{M}{k}} \cdot \frac{v_0}{2} = \frac{b}{2}.$$

So $X_{max} = l + \frac{l}{2}$

c) When the wheel hits the gear again, we again find the velocity. It turns out to be zero now.
So the wheel stops.