1 (3.4) Exploding projectile

By momentum conservation we know that the momenta of the two pieces are related to the momentum of the projectile at the point of explosion by:

\[ m \vec{v}_0 = m_1 \vec{v}_1 + m_2 \vec{v}_2, \quad (1) \]

where \( m_1 \) and \( m_2 = 3m_1 \) are the masses of the two pieces, and \( m = m_1 + m_2 = 4m_1 \) is the mass of the projectile.

Since \( m_1 \) returns to the launching station we conclude that \( \vec{v}_1 = -\vec{v}_0 \) (to get the reverse motion we just need to reverse the direction of the velocity). Choosing now our \( x \)-axis to be in \( \vec{v}_2 \) direction we have from (1):

\[ mv_0 = m_1 v_1 + m_2 v_2 = -m_1 v_0 + m_2 v_2 \]

\[ \therefore v_2 = v_0 \frac{m + m_1}{m_2} = \frac{5}{3} v_0. \quad (2) \]

Now the time for falling from the top point is the same as the time for getting to it (since the acceleration doesn’t depend on the mass). The time to get to the top point is \( T = L/v_0 \), thus the total distance the second mass will go is:

\[ L + v_2 T = L + \frac{5}{3} L = \frac{8}{3} L. \quad (3) \]

2 (3.5) Acrobat and a monkey

The velocity of the acrobat at height \( h \) would be determined from:

\[ h = \frac{v_0^2 - v^2}{2g} \]

\[ \therefore v = \sqrt{v_0^2 - 2gh} \quad (4) \]

assuming that \( 2gh \leq v_0^2 \), i.e. that the height \( h \) is reachable.
By momentum conservation, at the moment acrobat grabs the monkey:

\[ M \vec{v} = (M + m) \vec{v}', \]

where \( \vec{v}' \) is the velocity of the acrobat just after he grabbed the monkey.

Since \( \vec{v} \) and \( \vec{v}' \) are in the same direction, we have:

\[ v' = \frac{M}{M + m} v = \frac{M}{M + m} \sqrt{v_0^2 - 2gh} \]

Finally the total height the pair will reach is:

\[ h + \frac{(v')^2}{2g} = h + \left( \frac{M}{M + m} \right)^2 \frac{v_0^2}{2g} - h \]

3 (3.8) Impulsive woman

The initial velocity of the woman is related to her jump height by:

\[ v = \sqrt{2gh}. \]

Thus, the impulse she receives from the ground is:

\[ F dt = P(dt) - P(0) = mv - 0 = m\sqrt{2gh}, \]

pointing straight up (\( t = 0 \) is chosen to be the moment she jumps, and \( dt \) is an infinitesimal time interval).

4 (3.12) Sand-spraying locomotive

Note that since the distance between the locomotive and the freight car doesn’t change, they move with the same velocity. Thus from the ground frame the velocity of the sand at time \( t \) is \( v(t) + u \), where \( v(t) \) is the velocity of the locomotive/freight car at time \( t \) and \( u \) is the velocity of the sand relative to the locomotive.

The rate at which sand is sprayed into the freight car is constant, therefore the mass of the freight car at time \( t \) is equal to \( m(t) = m_0 + \frac{dm_s}{dt} t \).

By momentum conservation we have:

\[ m(t)v(t) + [v(t) + u]dm_s = [m(t) + dm_s]v(t + dt) = [m(t) + dm_s][v(t) + \frac{dv}{dt} dt], \]

where \( v(t) \) and \( v(t + dt) \) are the velocities of the freight car at times \( t \) and \( t + dt \) correspondingly, and in the last equality we Taylor expanded \( v(t + dt) \) throwing away terms of order \((dt)^2\) or higher.

Simplifying we have (note that we can throw away \( dm_s dt \) since it’s a higher order, i.e. much smaller term):

\[ (v + u)dm_s = m \frac{dv}{dt} dt + dm_s v \]

\[ \therefore \frac{dm_s}{dt} = \left( m_0 + \frac{dm_s}{dt} t \right) \frac{dv}{dt} \]

\[ \therefore v(t) = u \frac{dm_s}{dt} \int_0^t \frac{dt}{m_0 + \frac{dm_s}{dt} t} = u \ln \left( 1 + \frac{dm_s}{dt} t \right) \]

\[ \therefore v(100s) = 5m/s \ln \left( 1 + \frac{10kg/s100s}{2000kg} \right) \approx 2.0m/s \]
5 (3.14) N jumps from a flatcar

(a) If all of them jump off at the same time, by momentum conservation we have:

\[(Nm)(u - v) = Mv,\]  \hspace{1cm} (12)

where \(v\) is the final velocity of the flatcar. Since the velocity of men with respect to the car is \(u\), their velocity with respect to the ground is \(u - v\), which is what we have on the left hand side. Thus \(v = u \frac{Nm}{M + Nm}\).

(b) Once again by momentum conservation, for the first jump we have:

\[0 = [M + (N - 1)m]v_1 - m(u - v_1),\]  \hspace{1cm} (13)

since after the first jump \(N - 1\) men are still standing on the flatcar. For the second jump:

\[[M + (N - 1)m]v_1 = [M + (N - 2)m]v_2 - m(u - v_2).\]  \hspace{1cm} (14)

Note that the flatcar and \(N - 1\) men are now moving with respect to the ground, thus the momentum on the left hand side.

Clearly for the \(k\)-th jump we’ll have:

\[\begin{align*}
[M + (N - k + 1)m]v_{k-1} &= [M + (N - k)m]v_k - m(u - v_k) \\
\therefore v_k &= v_{k-1} + \frac{mu}{M + (N - k + 1)m} \\
\therefore v &= \sum_{k=0}^{N-1} \frac{mu}{M + (N - k)m}
\end{align*}\]  \hspace{1cm} (15)

(c) From

\[u \frac{Nm}{M + Nm} = \sum_{k=0}^{N-1} \frac{mu}{M + Nm},\]  \hspace{1cm} (16)

it should be clear that the second case when they all jump together gives a larger final velocity. A physical argument for this is that if they all jump together the recoil mass, which is the mass of \(N\) men plus the mass of the flatcar is larger than the recoil mass in the second case, where it’s getting smaller with every other man jumping off the car (to get a relative velocity \(u\) each jumper must accelerate himself plus the car).

6 (3.16) Shooting out of the fire hydrant

By Newton’s second law, the reaction force is equal to:

\[F = \frac{dP_{\text{water}}}{dt} = \frac{d(m_{\text{water}}V_0)}{dt} = V_0 \frac{d(\rho_{\text{water}}V)}{dt} = \rho_{\text{water}}SV_0^2.\]  \hspace{1cm} (17)

In the last equality we used that in time \(dt\) the volume of the shot out water would be \(V = SdL\), with \(dL = V_0dt\).